Eur. Phys. J. B **52**, 333–336 (2006)
DOI: 10.1140/epjb/e2006-00297-3

THE EUROPEAN
PHYSICAL JOURNAL B

Quantum controlled phase gate and cluster states generation via two superconducting quantum interference devices in a cavity

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Received 25 April 2006 / Received in final form 6 July 2006
Published online 31 July 2006 –

Abstract. A scheme for implementing 2-qubit quantum controlled phase gate (QCPG) is proposed with two superconducting quantum interference devices (SQUIDs) in a cavity. The gate operations are realized within the two lower flux states of the SQUIDs by using a quantized cavity field and classical microwave pulses. Our scheme is achieved without any type of measurement, does not use the cavity mode as the data bus and only requires a very short resonant interaction of the SQUID-cavity system. As an application of the QCPG operation, we also propose a scheme for generating the cluster states of many SQUIDs.

PACS. 03.67.Lx Quantum computation – 03.65.Ud Entanglement and quantum nonlocality – 42.50.Dv Nonclassical states of the electromagnetic field, including entangled photon states; quantum state engineering and measurements

1 Introduction

Recently, much attention has been paid to the realization of quantum computers [1], which could compete in certain tasks that classical computer could never fulfill in acceptable times [2]. Despite the already rather advanced theoretical concepts of quantum computing, the development of its physical implementations is just at an early stage. Up to now, many physical systems have been suggested as possible realizations of qubits and quantum logic gates [3]. In some mature systems quantum manipulations of a few qubits have already been demonstrated experimentally, such as cavity QED [4], trapped ions [5], nuclear magnetic resonance (NMR) [6] and photonic systems [7]. Despite recently exciting experimental progresses, the physical realization of a scalable quantum computer remains a great challenge. The building blocks of quantum computers are two-qubit logic gates. Few systems have demonstrated controlled qubits and qubit coupling between pairs taken from more than four qubits. It is difficult to couple different subsystems in a controlled manner, while at the same time shielding the system from the influence of its environment. One particularly attractive possibility is to use matter qubits to serve as hardware because they are static and potentially long lived, and an optical coupling mechanism can creates suitable entanglement. Among the variety of systems being explored for hardware implementations of quantum computers, cavity QED system is favored because of its demonstrated advantage when subjected to coherent manipulations [4]. However, in the cavity-atom system the strong coupling limit is difficult to meet and individual addressing of the particles is also a problem-maker. In contrast, the strong coupling limit was realized with superconducting charge [8] and flux qubits [9] in a microcavity. SQUIDs, as well as other solid state circuits, can be perfectly fixed, easily embedded in a cavity and easy to scale up, thus the cavity-SQUID scheme may be more preferable than cavity-atom system. In addition, by placing SQUIDs into a superconducting cavity, the external environment induced decoherence can be greatly suppressed because the cavity can serve as magnetic shield for SQUIDs.

It is well-known that single-qubit rotations and 2-qubit controlled logic gates together can be served to realize any unitary operation on n qubits [10]. Recently, people have presented various methods for implementing quantum logic operation via SQUID flux qubits. Zhou et al. [11] proposed a scheme to implement a single-qubit operation with a three-level Λ-type rf-SQUID qubit, which proven to be more favorable than the conventional two-level qubit. Amin et al. [12] gave a more general method to implement an arbitrary qubit rotation using the three-level qubit. However, under their assumption of small detuning, population in the upper level of the qubit is significant during the interaction, which results in higher probability of spontaneous decay, and hence gate errors. Yang et al. [13] proposed an alternative method for implementing arbitrary single-qubit operations with a three-level

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SQUID without energy relaxation. The above works [11–13] have proposed some schemes for realizing single-qubit operation and two-qubit entanglement, but the building blocks of quantum computers are two-qubit logic gates. Very recently, Song et al. [14] realized the 2-qubit QCPG operation by using a quantized cavity field and classical microwave pulses via Raman transition. In their scheme three SQUID qubits were used to perform the QCPG operation, one of the SQUID qubits is utilized as a quantum data-bus, and the gate operation is achieved between the remaining two SQIUID qubits. Yang et al. [15] proposed their scheme for realizing the n-qubit ($n \geq 2$) QCPG operation by using SQUIDs coupled to a superconducting resonator, where data-bus qubit of the gate operation is not needed. Enlightened by their progress, we report alternative ways of demonstrating 2-qubit QCPG operation via SQUIDs in a quantized superconducting cavity. Our scheme is achieved without any type of measurement, does not use the cavity mode as the data bus and only requires a very short resonant interaction.

In most of the current explorations, quantum logic gates are implemented with sequences of controlled interactions between selected particles. Recently, Raussendorf and Briegel [16] proposed a different model of a scalable quantum computer, namely the one way quantum computer, which constructs quantum logic gates by single-qubit measurements on cluster states. Thus many quantum computation scheme based on the cluster states have been proposed [17–19]. The distinct advantage of one-way computing strategy is that it separates the process of generating entanglement and executing the computation. So one can tolerate failures during the generation process simply by repeating the process providing the failures are heralded. On the other hand, due to its novel application in quantum computing, generation of the cluster states [20–25] also attracts many attentions. Here, as a direct application of the proposed 2-qubit QCPG operation, we also present a way of generating the multipartite cluster states via SQUIDs trapped in cavity.

2 Basic models

The SQUID considered in this paper is the radio frequency SQUID (rf-SQUID), which consisting of a Josephson tunnel junction in a superconducting loop, the size of which is on the order of 10–100 μm. The rf-SQUID considered here has three-level Λ-type energy structure, which includes two lower flux states $|0\rangle$, $|1\rangle$ and an upper state $|e\rangle$. The two lower flux states $|0\rangle$ and $|1\rangle$ reside in two distinct potential valley and serve as logic 0 and 1 in our proposal. Suppose the coupling of $|0\rangle$, $|1\rangle$ and $|e\rangle$ with other levels of the rf-SQUID via the cavity is negligible, which can be readily satisfied by adjusting the level spacings of the rf-SQUID. For a rf-SQUID, the level spacings can be easily changed by varying the external flux Φ_x or the critical current I_c [26]. Hence, coupling between microwave pulse and any particular rf-SQUID qubit can be obtained selectively, via frequency matching.

2.1 SQUID driven by classical field

We firstly review the way of implementing single-qubit operations, which can be realized by a rf pulse on the rf-SQUID. Let's firstly consider a rf-SQUID driven by a rf pulse. If it is resonant with the $|0\rangle \leftrightarrow |1\rangle$ transition but far-off resonant with the transitions of $|0\rangle \leftrightarrow |e\rangle$ and $|1\rangle \leftrightarrow |e\rangle$ of the rf-SQUID, the interaction Hamiltonian is [27]

$$H_I = i\Omega(|0\rangle\langle 1| - |1\rangle\langle 0|), \tag{1}$$

where Ω is the Rabi frequency between the levels $|0\rangle$ and $|1\rangle$. Thus a rf pulse with duration time t results in the following rotation

$$\begin{aligned} |0\rangle &\rightarrow \cos\Omega t|0\rangle - \sin\Omega t|1\rangle, \\ |1\rangle &\rightarrow \cos\Omega t|1\rangle + \sin\Omega t|0\rangle. \end{aligned} \tag{2}$$

Similarly, single-qubit operations in the basis $\{|0\rangle, |e\rangle\}$ and $\{|1\rangle, |e\rangle\}$ can also be realized. In the rest part of this letter, we will use some single qubit operations without specifying the interaction time.

2.2 Cavity-SQUIDs resonant interaction

We then consider 2 rf-SQUID qubits simultaneously interacting with a single-mode cavity. The distance of the two qubits is large enough so that the interaction between the two SQUIDs is completely negligible. If the cavity mode is resonant with the $|0\rangle \leftrightarrow |1\rangle$ transition but far-off resonant with the transitions of $|0\rangle \leftrightarrow |e\rangle$ and $|1\rangle \leftrightarrow |e\rangle$ of the 2 rf-SQUIDs, the interaction Hamiltonian can be expressed, in the interaction picture [27], as

$$H = \Omega_1(a^+|0\rangle_1\langle 1| + a|1\rangle_1\langle 0|) + \Omega_2(a^+|0\rangle_1\langle 1| + a|1\rangle_1\langle 0|), \tag{3}$$

where Ω_1 and Ω_2 are the coupling strength of the SQUIDs 1 and 2 with the cavity, respectively. a^+ and a are the creation and annihilation operators for the cavity mode. Then we can obtain the following evolutions

$$\begin{aligned} |0\rangle_1|0\rangle_2|0\rangle &\rightarrow |0\rangle_1|0\rangle_2|0\rangle, \\ |0\rangle_1|e\rangle_2|0\rangle &\rightarrow |0\rangle_1|e\rangle_2|0\rangle, \\ |1\rangle_1|0\rangle_2|0\rangle &\rightarrow \frac{\Omega_1}{\Omega}\left[\frac{1}{\Omega}\left(\Omega_1\cos\Omega t + \frac{\Omega_2^2}{\Omega_1}\right)|1\rangle_1|0\rangle_2|0\rangle\right. \\ &\quad \left. + \frac{\Omega_2}{\Omega}(\cos\Omega t - 1)|0\rangle_1|1\rangle_2|0\rangle - i\sin\Omega t|0\rangle_1|0\rangle_2|1\rangle\right], \\ |1\rangle_1|e\rangle_2|0\rangle &\rightarrow (\cos\Omega_1 t|1\rangle_1|0\rangle - i\sin\Omega_1 t|0\rangle_1|1\rangle)|e\rangle_2, \end{aligned} \tag{4}$$

where $\Omega = \sqrt{\Omega_1^2 + \Omega_2^2}$.

3 QCPG operation

Now, we consider the implementation of the 2-qubit QCPG.

Step 1. Let qubit 2 interact with a classical fields tuned to the transition $|1\rangle \leftrightarrow |e\rangle$. Choosing the amplitude and phase of the classical field appropriately, one obtains the transition $|1\rangle_2 \longrightarrow |e\rangle_2$.
Step 2. Send the two SQUID qubits simultaneously to the vacuum cavity. If we choose $\Omega_1 t = \pi$ and $\Omega_2 = \sqrt{3}\Omega_1$, which can be achieved by choosing coupling strengths and interaction time appropriately. Then we have

$$\begin{aligned} &|0\rangle_1|0\rangle_2 \rightarrow |0\rangle_1|0\rangle_2, |0\rangle_1|e\rangle_2 \rightarrow |0\rangle_1|e\rangle_2,\\ &|1\rangle_1|0\rangle_2 \rightarrow |1\rangle_1|0\rangle_2, |1\rangle_1|e\rangle_2 \rightarrow -|1\rangle_1|e\rangle_2, \end{aligned} \quad (5)$$

where we have omitted the cavity state, which was left in the vacuum state.
Step 3. Let qubit 2 interact again with a classical field tuned to the transition $|1\rangle \leftrightarrow |e\rangle$ and one obtains the transition $|e\rangle_2 \longrightarrow |1\rangle_2$.

The states of the system after each of the above steps can be described as

$$\begin{aligned} &|0\rangle_1|0\rangle_2 \longrightarrow |0\rangle_1|0\rangle_2 \longrightarrow |0\rangle_1|0\rangle_2 \longrightarrow |0\rangle_1|0\rangle_2,\\ &|0\rangle_1|1\rangle_2 \longrightarrow |0\rangle_1|e\rangle_2 \longrightarrow |0\rangle_1|e\rangle_2 \longrightarrow |0\rangle_1|1\rangle_2,\\ &|1\rangle_1|0\rangle_2 \longrightarrow |1\rangle_1|0\rangle_2 \longrightarrow |1\rangle_1|0\rangle_2 \longrightarrow |1\rangle_1|0\rangle_2,\\ &|1\rangle_1|1\rangle_2 \longrightarrow |1\rangle_1|e\rangle_2 \longrightarrow -|1\rangle_1|e\rangle_2 \longrightarrow -|1\rangle_1|1\rangle_2. \end{aligned} \quad (6)$$

The above transformations correspond to 2-qubit QCPG operation, where qubit 1 and 2 are the control and target qubit, respectively. In this way, a scheme for implementing the 2-qubit QCPG based on cavity-SQUID system is proposed. Our scheme is achieved without any type of measurement, does not use the cavity mode as the data bus and only requires a single resonant interaction of the SQUID-cavity system. Thus the presented scheme is very simple and the required interaction time is very short. The simplification of the process and the reduction of the operation time are important for suppressing decoherence.

4 Generation of cluster states

Now, we consider the generation of the cluster states. Firstly, we focus on the 2-qubit cluster state case. Initially, rf-SQUIDs 1 and 2 have been prepared in the state $1/\sqrt{2}(|0\rangle + |1\rangle)$ and the cavity in vacuum state $|0\rangle_C$. Thus the state of the quantum system is

$$\frac{1}{2}(|0\rangle_1 + |1\rangle_1)(|0\rangle_2 + |1\rangle_2)|0\rangle_C. \quad (7)$$

Repeat the process of the above 2-qubit QCPG operation on rf-SQUIDs 1 and 2 with the aid of the cavity. After the gate operation the state of the quantum system will evolve to

$$\frac{1}{2}(|0\rangle_1 + |1\rangle_1\sigma_1)(|0\rangle_2 + |1\rangle_2), \quad (8)$$

where $\sigma_1 = |0\rangle_2\langle 0| - |1\rangle_2\langle 1|$ and we have omitted the state of the cavity, which is left in the vacuum state and disentangled with the prepared entangled state. The state in equation (8) is a bipartite cluster state.

Next, we consider the generation of N-qubit cluster state. Assume all the N rf-SQUIDs have been prepared in the state $1/\sqrt{2}(|0\rangle + |1\rangle)$ and a cavity in the vacuum state. the initial state of the system is

$$\frac{1}{\sqrt{2^N}} \bigotimes_{i=1}^{N} (|0\rangle_i + |1\rangle_i)|0\rangle_C. \quad (9)$$

For rf-SQUID i ($i < N$), repeat the above procedures for generating bipartite cluster state on the rf-SQUIDs i, $(i + 1)$ and the cavity (these selective interactions can be embodied via frequency matching) and end the process when $i = N$. Then the N rf-SQUIDs will be prepared in

$$\frac{1}{\sqrt{2^N}} \bigotimes_{i=1}^{(N-1)} (|0\rangle_i + |1\rangle_i\sigma_i)(|0\rangle_N + |1\rangle_N), \quad (10)$$

with $\sigma_i = |0\rangle_{(i+1)}\langle 0| - |1\rangle_{(i+1)}\langle 1|$. The state in (10) is the N-qubit cluster state and we have omitted the state of the cavity, which is disentangled with the cluster state.

For the case of the inevitable imperfectness of the experimental exploration of our scheme, it is very reasonable that one can only generate the cluster state of a certain length we label this critical number as n_c. To generate a cluster chain of a length $n > n_c$, we can simply parallelling generate cluster chains of length under the critical number and then fuse them together to further increase its length. This idea can also be perfected even if the QDPG is not deterministically [23]. In reference [23], Duan et al. also generalized the idea to the generation of two-dimensional square lattice cluster states from a set of cluster chains with QCPG only succeed with an arbitrarily small probability. The two-dimensional square lattice cluster state prepared at numerous qubits is a universal "substrate" for quantum computation. After the preparation of the states, the remaining work is only to perform single-qubit measurements, and the final results are read out from those qubits that were not measured in the whole process. In this sprit, many recent schemes [23, 25, 28] have shown how to use probabilistic gate operations to construct entangled states with certainty. Our scheme is of deterministic nature, thus it holds more promising future.

5 Discussions and conclusion

Before ending the paper, we briefly address the experimental feasibility of the proposed scheme. Among the variety of systems being explored for hardware implementations for quantum computation, the cavity-SQUID system, as well as other solid state circuits, is favored because it is easy embedded in electronic circuits and scaled up to large registers [29], and the control and measurement techniques are quite advanced [30]. The interaction time can be perfectly controlled by external control. The time constants involved are long enough to realize all the required manipulations [9]. Finally, coupling between microwave pulses and any particular rf-SQUID qubit can be obtained selectively via frequency matching. Thus our scheme might be realizable within current technology.

For the sake of definitiveness, let us estimate the experimental feasibility of realizing the logic gates using SQUIDs with the parameters already available in present experiment [31–33]. Suppose the quality factor of the superconducting cavity is $Q = 1 \times 10^6$ and the cavity mode frequency is $\omega_c = 50$ GHz, the cavity decay time is $k^{-1} = Q/\omega_c = 20$ μs. The realistic length of the cavity is $l = 1.4$ mm, when the two SQUIDs are located each at one of the antinodes of the cavity, the distance of them, D, is equal to the length of the cavity. For SQUIDs of size $d = 40$ μm, we get $D/d = 35$ which perfectly satisfies the requirement of $D \gg d$. The upper state energy relaxation constant is $\gamma_e^{-1} = 2.5$ μs. For a superconducting standing-wave cavity and a SQUID located at one of antinodes of the magnetism field, the coupling constant is $g = 1.8 \times 10^8$ Hz, thus the resonant interaction time for our scheme is $T_r = \pi/g \simeq 1.7 \times 10^{-8}$ s. Meanwhile, the strong coupling condition can be perfectly achieved as $g^2/\gamma_e k = 1.6 \times 10^6 \gg 1$. The Rabi frequency for the interaction of the rf pulse and the SQUID is $\Omega = 8.5 \times 10^7$ Hz, thus the interaction time for the single photon detection is $T_l = \pi/2\Omega \simeq 1.8 \times 10^{-8}$ s. Both interaction time (T_r and T_l) are much shorter than the cavity decay time and the relaxation time of the upper state. In addition, the above estimation is very conservative compare to present experimental achievements, in fact, a superconducting cavity with $Q = 3 \times 10^8$ has already been reported [34].

In conclusion, we have investigated a simple scheme for implementing the 2-qubit QCPG gate based on cavity-SQUID systems. The presented schemes are achieved without any type of measurement. Our scheme is also a deterministic one without any auxiliary SQUID serving as data-bus. In addition, the implementation of our scheme is simple, which is very important in view of decoherence and the successful probability and the fidelity both reach unit. We also roughly estimated the experimental feasibility of our schemes, which shows they are well within current techniques, thus our suggestion may offer a simple way of implementing the quantum logic gate via QED-SQUID system. As an direct application of the QCPG operation, we also proposed a scheme for generating the cluster states, which are universal “substrate” for quantum computation.

This work is supported by the Key Program of the Education Department of Anhui Province (No. 2006kj070A), the Talent Foundation of Anhui University and for Z.-Y. Xue by the Postgraduate Innovation Research Plan from Anhui University.

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